ΤΥΠΟΛΟΓΙΟ ΜΑΘΗΜΑΤΙΚΩΝ Α΄ ΛΥΚΕΙΟΥ

Δ Υ Ν Α Μ Ε Ι Σ

$\alpha^\nu = \underbrace{\alpha\cdot\alpha\cdots\alpha}_{\nu-\pi\alpha\rho\alpha\gamma o\nu\tau\varepsilon\varsigma}$, $\alpha^0 = 1, \alpha \neq 0, \alpha^1 = \alpha$

$\alpha^\mu \cdot \alpha^\nu = \alpha^{\mu+\nu}$, $\dfrac{\alpha^\mu}{\alpha^\nu} = \alpha^{\mu-\nu}$, $(\alpha^\mu)^\nu = \alpha^{\mu\cdot\nu}$

$(\alpha\cdot\beta)^\nu = \alpha^\nu \cdot \beta^\nu$, $\left(\dfrac{\alpha}{\beta}\right)^\nu = \dfrac{\alpha^\nu}{\beta^\nu}$, $\alpha^{-\nu} = \dfrac{1}{\alpha^\nu}, \alpha \neq 0$

$\left(\dfrac{\alpha}{\beta}\right)^{-\nu} = \left(\dfrac{\beta}{\alpha}\right)^\nu$, $\alpha, \beta \neq 0$

Τ Α Υ Τ Ο Τ Η Τ Ε Σ

$(\alpha \pm \beta)^2 = \alpha^2 \pm 2\alpha\beta + \beta^2$, $(\alpha+\beta)(\alpha-\beta) = \alpha^2 - \beta^2$

$(\alpha \pm \beta)^3 = \alpha^3 \pm 3\alpha^2\beta + 3\alpha\beta^2 \pm \beta^3$, $\alpha^3 \pm \beta^3 = (\alpha \pm \beta)(\alpha^2 \mp \alpha\beta + \beta^2)$

$(\alpha+\beta+\gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta + 2\beta\gamma + 2\alpha\gamma$

$\alpha^\nu - \beta^\nu = (\alpha-\beta)(\alpha^{\nu-1} + \alpha^{\nu-2}\beta + \cdots + \alpha\beta^{\nu-2} + \beta^{\nu-1})$, ν φυσικός

$\alpha^\nu + \beta^\nu = (\alpha+\beta)(\alpha^{\nu-1} - \alpha^{\nu-2}\beta + \cdots - \alpha\beta^{\nu-2} + \beta^{\nu-1})$, ν περιττός

Α Π Ο Λ Υ Τ Η Τ Ι Μ Η

$|\alpha| = \begin{cases} \alpha, \alpha \geq 0 \\ -\alpha, \alpha < 0 \end{cases}$, $|\alpha| \geq 0, -\alpha \leq |\alpha| \leq \alpha$, $|\alpha|^2 = \alpha^2, |\alpha| = |-\alpha|$

Αν $\theta \geq 0$ τότε ισχύει $|x| = \theta \Leftrightarrow x = \theta$ η $x = -\theta$

$|x| = |\theta| \Leftrightarrow x = \theta$ η $x = -\theta$, $\sqrt{\alpha^2} = |\alpha|$

$|x| \leq \theta \Leftrightarrow -\theta \leq x \leq \theta$, $\theta \geq 0$

$|x| \geq \theta \Leftrightarrow x \geq \theta$ η $x \leq -\theta$, $\theta \geq 0$

$|\alpha\cdot\beta| = |\alpha|\cdot|\beta|$, $\left|\dfrac{\alpha}{\beta}\right| = \dfrac{|\alpha|}{|\beta|}, \beta \neq 0$, $\big||\alpha| - |\beta|\big| \leq |\alpha+\beta| \leq |\alpha| + |\beta|$

Ρ Ι Ζ Ε Σ

$\sqrt[\nu]{\alpha} = x \Leftrightarrow x^\nu = \alpha$, όπου $\alpha \geq 0$, $x \geq 0$, $\nu \in \mathbb{N}$

$\left(\sqrt[\nu]{\alpha}\right)^\nu = \sqrt[\nu]{\alpha^\nu} = \alpha$, $\alpha \geq 0$ $\sqrt{\alpha^2} = |\alpha|$ όπου $\alpha \in \mathbb{R}$

Αν $\alpha, \beta \succ 0$: $\sqrt[\nu]{\alpha}\cdot\sqrt[\nu]{\beta} = \sqrt[\nu]{\alpha\beta}$, $\dfrac{\sqrt[\nu]{\alpha}}{\sqrt[\nu]{\beta}} = \sqrt[\nu]{\dfrac{\alpha}{\beta}}$, $\sqrt[\nu]{\sqrt[\mu]{\alpha}} = \sqrt[\nu\mu]{\alpha}$

$\left(\sqrt[\nu]{\alpha}\right)^\kappa = \sqrt[\nu]{\alpha^\kappa}$, $\sqrt[\nu]{\alpha^\kappa} = \sqrt[\nu\rho]{\alpha^{\kappa\rho}}$, $\sqrt[\nu]{\alpha^\kappa} = \alpha^{\frac{\kappa}{\nu}}$, $\sqrt[\nu]{\alpha^\nu\beta} = \alpha\sqrt[\nu]{\beta}$

$x^\nu = \alpha \Rightarrow \left\{\begin{array}{l} \nu : \alpha\rho\tau\iota o\varsigma \rightarrow x = \pm\sqrt[\nu]{\alpha}, o\tau\alpha\nu : \alpha \geq 0, \alpha\delta\upsilon\nu\alpha\tau\eta, o\tau\alpha\nu : \alpha < 0 \\ \nu : \pi\varepsilon\rho\iota\tau\tau o\varsigma \rightarrow x = \sqrt[\nu]{\alpha}, o\tau\alpha\nu : \alpha \geq 0, x = -\sqrt[\nu]{-\alpha}, o\tau\alpha\nu : \alpha < 0 \end{array}\right\}$

$x^\nu = \alpha^\nu \Rightarrow \begin{cases} \alpha\nu, \nu : \alpha\rho\tau\iota o\varsigma, \tau o\tau\varepsilon \rightarrow x = \pm\alpha \\ \alpha\nu, \nu : \pi\varepsilon\rho\iota\tau\tau o\varsigma, \tau o\tau\varepsilon \rightarrow x = \alpha \end{cases}$

ΠΛΗΘΟΣ ΡΙΖΩΝ ΤΗΣ ΕΞΙΣΩΣΗΣ $\alpha x^2+\beta x+\gamma=0, \alpha\neq 0$

ΔΙΑΚΡΙΝΟΥΣΑ : $\Delta=\beta^2-4\alpha\gamma$	
Αν $\Delta>0$	Έχει δύο ρίζες άνισες $x_{1,2}=\dfrac{-\beta\pm\sqrt{\Delta}}{2\alpha}$
Αν $\Delta=0$	Έχει μια διπλή ρίζα $x_1=x_2=\dfrac{-\beta}{2\alpha}$
Αν $\Delta<0$	Δεν έχει πραγματικές ρίζες

ΤΥΠΟΙ ΤΟΥ Vieta

Αν x_1, x_2 ρίζες της εξίσωσης $\alpha x^2+\beta x+\gamma=0, \alpha\neq 0$, τότε

$S=x_1+x_2=-\dfrac{\beta}{\alpha}$, $P=x_1\cdot x_2=\dfrac{\gamma}{\alpha}$, $x^2-Sx+P=0$

ΠΡΟΣΗΜΟ ΤΗΣ $f(x)=\alpha x^2+\beta x+\gamma, \alpha\neq 0, x\in \mathbb{R}$

Αν $\Delta>0$	$-\infty$ $\quad x_1$	x_2	$+\infty$
$F(x)$	0μόσημο του α	Ετερόσημο του α	Ομόσημο του α
Αν Δ=0	$-\infty$	-β/2α	$+\infty$
$F(x)$	Ομόσημο του α	Ομόσημο του α	
Αν $\Delta<0$	$-\infty$		$+\infty$
$F(x)$	Ομόσημο του α		

ΣΥΣΤΗΜΑ 2x2

$\begin{cases}\alpha x+\beta y=\gamma \\ \alpha' x+\beta' y=\gamma'\end{cases}$ $D=\begin{vmatrix}\alpha & \beta \\ \alpha' & \beta'\end{vmatrix}$, $D_x=\begin{vmatrix}\gamma & \beta \\ \gamma' & \beta'\end{vmatrix}$, $D_y=\begin{vmatrix}\alpha & \gamma \\ \alpha' & \gamma'\end{vmatrix}$

Αν $D\neq 0$ έχει μοναδική λύση $x=\dfrac{D_x}{D}$, $y=\dfrac{D_y}{D}$

Αν $D=0$: $\begin{cases} D_x\neq 0, \eta, D_y\neq 0 \rightarrow \text{ΑΔΥΝΑΤΟ} \\ D_x=D_y=0, \begin{cases}\text{ΑΟΡΙΣΤΟ} \\ \text{ΑΔΥΝΑΤΟ}, \alpha\nu: \alpha=\alpha'=\beta=\beta'=0, \kappa\alpha\iota, \gamma\neq 0, \eta, \gamma'\neq 0\end{cases}\end{cases}$

Τ Ρ Ι Γ Ω Ν Ο Μ Ε Τ Ρ Ι Α

$\eta\mu^2\theta+\sigma\upsilon\nu^2\theta=1$, $\varepsilon\phi\theta=\dfrac{\eta\mu\theta}{\sigma\upsilon\nu\theta}$, $\sigma\phi\theta=\dfrac{\sigma\upsilon\nu\theta}{\eta\mu\theta}$, $\varepsilon\phi\theta\cdot\sigma\phi\theta=1$

$\eta\mu(-x)=-\eta\mu(x)$, $\eta\mu(2\kappa\pi+x)=\eta\mu x$, $\eta\mu(\pi\pm x)=\mp\eta\mu x$, $\eta\mu\left(\dfrac{\pi}{2}\pm x\right)=\sigma\upsilon\nu x$

$\sigma\upsilon\nu(-x)=\sigma\upsilon\nu x$, $\sigma\upsilon\nu(2\kappa\pi+x)=\sigma\upsilon\nu x$, $\sigma\upsilon\nu(\pi\pm x)=-\sigma\upsilon\nu x$, $\sigma\upsilon\nu\left(\dfrac{\pi}{2}\pm x\right)=\mp\eta\mu x$

$\varepsilon\phi(-x)=-\varepsilon\phi x$, $\varepsilon\phi(2\kappa\pi+x)=\varepsilon\phi x$, $\varepsilon\phi(\pi\pm x)=\pm\varepsilon\phi x$, $\varepsilon\phi\left(\dfrac{\pi}{2}\pm x\right)=\mp\sigma\phi x$

$\sigma\phi(-x)=-\sigma\phi x$, $\sigma\phi(2\kappa\pi+x)=\sigma\phi x$, $\sigma\phi(\pi\pm x)=\pm\sigma\phi x$, $\sigma\phi\left(\dfrac{\pi}{2}\pm x\right)=\mp\varepsilon\phi x$